THE CENTRE FOR INDEPENDENT STUDIES

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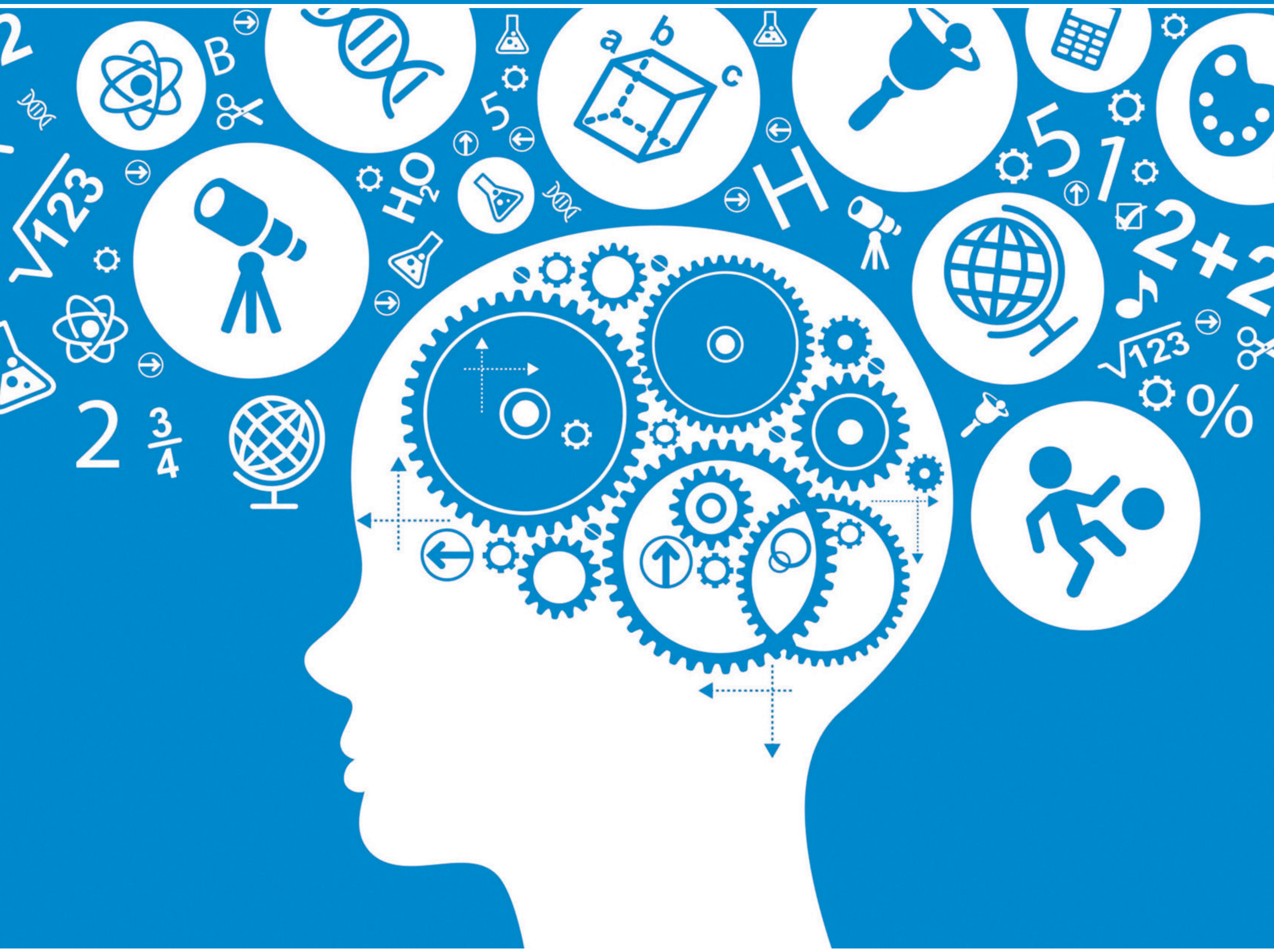

Setting the preschool foundation for success in Mathematics

David C. Geary

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Contents

Executive Summary

- The mathematical competencies that children develop in school have a long-term influence on their employability and wages in adulthood, as well as on their ability to navigate the many quantitative demands of day-to-day life in the modern world.
- Children who start school behind their peers in this 'number knowledge' are likely to stay behind throughout schooling and into adulthood.
- The foundation for this school-entry number knowledge in turn emerges during the preschool years, meaning that many children are already at-risk of long-term difficulties with mathematics by 4 years of age.
- The core early quantitative knowledge that undergirds school readiness appears to be children's learning of the count list (i.e., the ability to count, "one, two, three…"), using counting to enumerate (i.e., determine how many) collections of objects, and especially their conceptual understanding of the magnitudes represented by number words and numerals.
- Children who lag in the conceptual understanding of the cardinal value of number words start school substantively behind their earlier developing peers, controlling for parental education, verbal and nonverbal intelligence, executive functions, and other factors.
- Parent-child interactions at home facilitate some aspects of early quantitative development, such as learning number words, but most of these activities do not promote the development of cardinal knowledge. Further work is needed to better understand how the home environment fosters the development of this conceptual knowledge.
- Moving forward will likely require the development of multi-systemic interventions that target parent-child number-related activities, preschool experiences, and child-centred factors (e.g., to promote better attentive behaviour in classroom settings).

Introduction

Mathematics as a scientific and applied field slowly emerged over the past several millennia and is now one of humanity's crowning achievements.[1] The historical recency and evolutionary novelty of mathematics make it a uniquely human competency and, unfortunately, one that is difficult to acquire.[2] A fuller understanding of the factors that contribute to mathematical development is critical, because success in the modern world depends to some extent on the mathematical competencies that students develop during schooling. These competencies create opportunities to pursue math-intensive careers in college and beyond, and have a long-term influence on employability and wages across many occupations, as well as an influence on the ease of coping with the day-to-day quantitative tasks of daily life.[3]

It is now well-documented that students who start school behind their peers in basic mathematics skills are at high risk of remaining behind throughout schooling and into adulthood.[4] For instance, Ritchie and Bates found that mathematics achievement at 7 years of age predicted occupational status 35 years later, controlling for the economic status of the family of origin, intelligence, reading achievement, and other factors. Mathematics achievement at 7 years, in turn, is predicted by earlier quantitative knowledge and skills; making the basic quantitative competencies that emerge during the preschool years foundational to formal learning in school.

However, there are many early quantitative competencies, ranging from learning count words ('one, two, three…') to the names of shapes, and thus the most essential ones are not clear. Identifying the core early competencies that support formal mathematics learning at school entry will facilitate the identification of children who are at risk for later learning difficulties and provide direction for the development of interventions to ameliorate these risks.

The goal here is to overview the results of a four-year longitudinal study — including two years of preschool (age 4 years), kindergarten, and first grade — designed to identify the early quantitative competencies that predict readiness to learn mathematics at school entry. The key school-entry competencies include number system knowledge: that is, a network of associations among number words and numerals, understanding their relative magnitudes, and arithmetically operating on these magnitudes. This knowledge supports the ability to quickly and accurately determine the larger of two numerals (e.g., 8 > 5), use counting to solve arithmetic problems, retrieve arithmetic facts (e.g., 2 + 3 = 5) from long-term memory, decompose numerals into equal sets [e.g., 7 = (6 + 1) = (5 + 2), etc.], and related skills.

In the early grades, performance on these types of measures is consistently related to concurrent and later mathematics achievement.[5] Moreover, school-entry number system knowledge predicts later mathematical competencies that contribute to employability and wages in young adults; after controlling for general cognitive ability, working memory, standardised mathematics achievement, and ethnicity, among other factors.[6]

The first section provides a brief overview of the quantitative competencies that are found in infants and young children without formal or informal instruction and those that emerge during the preschool years. The second section focuses on the home experiences that seem to foster quantitative development. This is followed by a summary of implications from these findings, along with some brief conclusions.

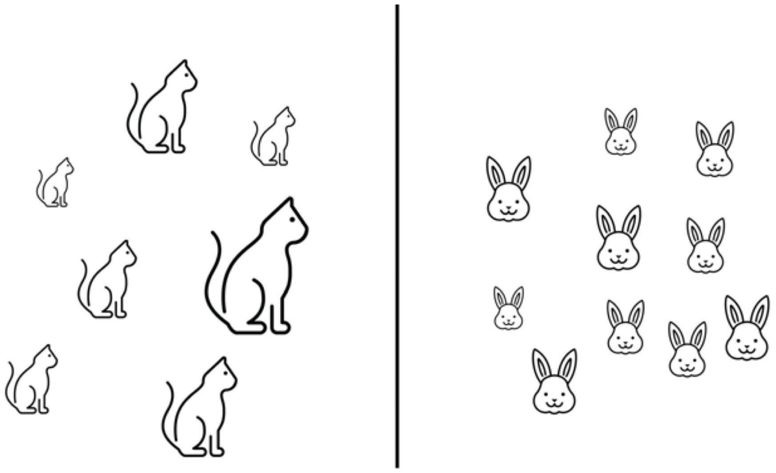
Early Quantitative Competencies

The development of a solid foundation for mathematics learning at school entry requires, as noted, identifying the core skills and knowledge that support this learning. There are two schools of thought on this matter: one focusing on people's intuitive sense of quantity, and the other focusing on the early acquisition of formal, symbolic knowledge (e.g., learning the sequence of number words and recognising numerals).

Intuitive Sense of Quantity

There are specific areas in the brain that generate magnitude representations of many features of the physical world, such as distance (e.g., closer to farther).[7] Quantitative information is one of these features and the corresponding abilities are supported by the approximate number system (ANS). The ANS is found in many species and is thought to be used in a variety of natural activities, such as finding the most abundant feeding spot.[8] The ANS aligns quantities along a type of mental number line that allows comparisons.[9] For instance, quickly glance at Figure 1 and determine if there are more cats or more rabbits without counting them. Most people will correctly determine there are more rabbits, even though the largest animals are cats and the total area covered by the cats is larger than that covered by the rabbits. The ease of discriminating one quantity from another depends on the ratio between them and not their absolute difference, such that discriminating 7 from 10 objects (1.43 ratio) is easier than discriminating 27 from 30 objects (1.11 ratio).

Figure 1:



The approximate number system (ANS) provides an evolved and intuitive understanding of relative quantity.

Preschoolers and even infants can make similar discriminations, although the ratio needs to be larger than that discernable by adults.[10] This is important because it has been proposed that the ANS is the brain and cognitive seat of all later formal or symbolic mathematics learning.[11] If so, then training that directly improves the ANS — that is, makes quantity discriminations easier — should facilitate children's learning the quantities represented by number words, numerals, and even do some basic arithmetic. Indeed, quite a few interventions have been developed to improve the precision of the ANS, but whether this also improves symbolic mathematics learning is hotly debated among neuroscientists and cognitive scientists.[12] We found that preschoolers' ANS acuity contributed to their initial learning of the meaning of number words (more on this below), but once children understood this, the ANS was not as important for further mathematics learning.[13] The implication is that training the ANS will not have long-term benefits for children's mathematics learning, in keeping with the results of a recent meta-analysis (across many studies) of ANS intervention studies.[14]

Early Symbolic Number Knowledge

Even if the ANS supports the early emergence of some aspects of children's symbolic knowledge, it is not sufficient. Adults in traditional contexts can make discriminations of approximate quantities just as well as college students in Western countries, but they have little understanding of symbolic mathematics (e.g., understanding the quantities represented by numerals) unless they have had some formal schooling.[15] However, in Western contexts many young children acquire some early symbolic quantitative knowledge, although the informal experiences that promote this development are not fully understood; as described in the next section.

Whatever the experiences, there has been a lot of research on the development of young children's math-related skills and knowledge. This includes studies of their learning of the count string (i.e., the sequence of number words 'one, two, three…'), understanding of ordinal relationships (e.g., 3 > 2), use of counting to understand 'how many' and their acquisition of counting principles (e.g., tag each counted object with only one count word), understanding the quantities represented by number words (i.e., their cardinal values), recognising and ordering numerals, intuitive understanding of addition and subtraction, and learning the names and features of basic geometric shapes, among others.[16] Surprisingly, there is relatively little research that links these early-developing competencies with mathematics achievement in school.

In a series of studies, we sought to bridge these literatures by identifying the key early competencies that predict mathematics achievement during the preschool years and that provide the foundation for mathematics learning at the beginning of first grade.[17] Of the 12 quantitative competencies assessed in these studies, preschoolers' understanding of the quantities represented by number words — that is, their cardinal value — has consistently emerged as the key predictor of later mathematics outcomes. There are also several precursor skills, including learning the count string and how to use counting to determine quantity — enumeration — that must be in place before children understand the cardinal values of number words and numerals.

In hindsight, it is not too surprising that cardinal knowledge emerged as a consistent predictor of later mathematics achievement, given it is children's first conceptual understanding of symbolic mathematics, and much of their later mathematical development (e.g., addition) is dependent on an understanding of the quantities represented by number words and numerals.

In any case, children's acquisition of cardinal knowledge is a slow and effortful process, as illustrated by the give-a-number task.[18] In this task, children are asked to provide *x* number of toys to an experimenter from a pile of toys (e.g., hand me 'one' rabbit). One-knowers provide one object when asked to do so but random amounts for other number words. In the middle-class to upper-middle class samples assessed in most of these studies, one-knower status is typically achieved between 2 and 3 years of age and two-knower status emerges about 3–6 months later. It takes additional 6–12 months to become three- and then four-knowers. Sometime thereafter, children have the conceptual insight that each number word in their count list (i.e., the number words they know) represents a unique quantity and that successive numbers are one more than the number before it. These children are considered cardinal principle knowers, although it will take several more years before they understand that each successive number represents $n + 1$ *ad infinitum*, not just for the number words that they know.[19]

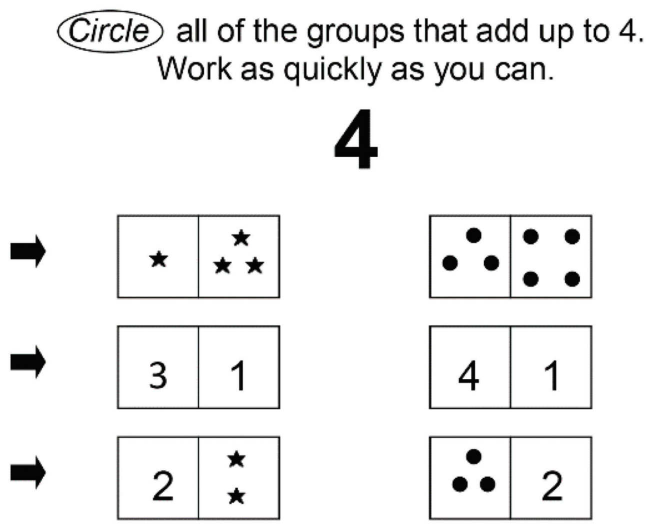
We assessed an economically and ethnically diverse group of young children multiple times during each of the two years of preschool, several times in kindergarten, and several times in first grade.[20] In one study, we sought to determine the beginning of preschool (mean age = 46 months) competencies that predicted end of preschool (mean age = 64 months) standardised mathematics achievement, controlling for age, sex, parental education, and intelligence. Children who had some of the count list memorised (e.g., they could count to 15), recognised several numerals, and performed well on the give-a-number task had mathematics achievement advantages at the end of preschool. For instance, children who started preschool as one-knowers had substantively lower mathematics achievement at the end of preschool relative to children who started as cardinal principle knowers. The gap was such that almost 3 out of 4 the one-knowers had lower mathematics achievement at the end of preschool than did the average cardinal principle knower, controlling other factors.[21]

This study and several related ones implicated cardinal knowledge as the foundation for later mathematics achievement. In fact, in one study we found that almost half of the variation in preschoolers' standardised mathematics achievement was explained by their beginning-of-preschool cardinal knowledge.[22] The long-term goal though was to predict their number system knowledge at the beginning of first grade. Here, we used the same number knowledge measures that predicted — better than early standardised mathematics achievement tests — middle school performance on quantitative measures used by labour economists to predict employability, wages, and productivity of young of adults.[23] These number system knowledge measures included the sophistication of the strategies used to solve arithmetic problems (e.g., finger counting vs. retrieval of facts from long-term memory), their understanding of the mathematical number line, and fluency of processing the quantities represented by numerals; the latter is illustrated in Figure 2.[24]

One core issue addressed in this study was whether the age at which children became cardinal principle knowers mattered for school readiness.[25] The assumption had been that children would be ready for formal mathematics learning, if they became cardinal principle knowers by the time they started kindergarten; that is, by about 5 years of age. This was not the case. About 10% of our sample were not cardinal principle knowers before the end of preschool (5 years and 2 months of age) and relative to these children, cardinal principle knowers at the beginning of preschool (3 years, 10 months) or children who achieved this status before the end of the first year (4 years, 2 months) had large advantages on the number system knowledge measures in early first grade (6 years, 9 months), controlling for parental education, verbal and nonverbal intelligence, executive functions, and other factors. The gap was such that almost 6 in 7 children who did not achieve cardinal principle status during preschool were below the mean of the early achievers in school readiness (as measured by the number system knowledge measures). We also found that age of achieving cardinal principle status was unrelated to later word reading achievement, indicating that it was not capturing broad academic achievement.

The question then became why early cardinal principle achievers had such a large advantage in later number system knowledge, after controlling for the myriad factors that typically predict individual differences in school readiness. We reasoned that children's learning of the relations among Arabic numerals begins only after they understand cardinality, and thus children who achieve this insight at a younger age have more time to elaborate their number system knowledge than do their later-developing peers. To test this hypothesis, we examined growth in children's knowledge of the relative magnitudes of Arabic numerals (e.g., determining which is larger: 5 or 3) before and after they understood the cardinality principle.[26]

Figure 2:

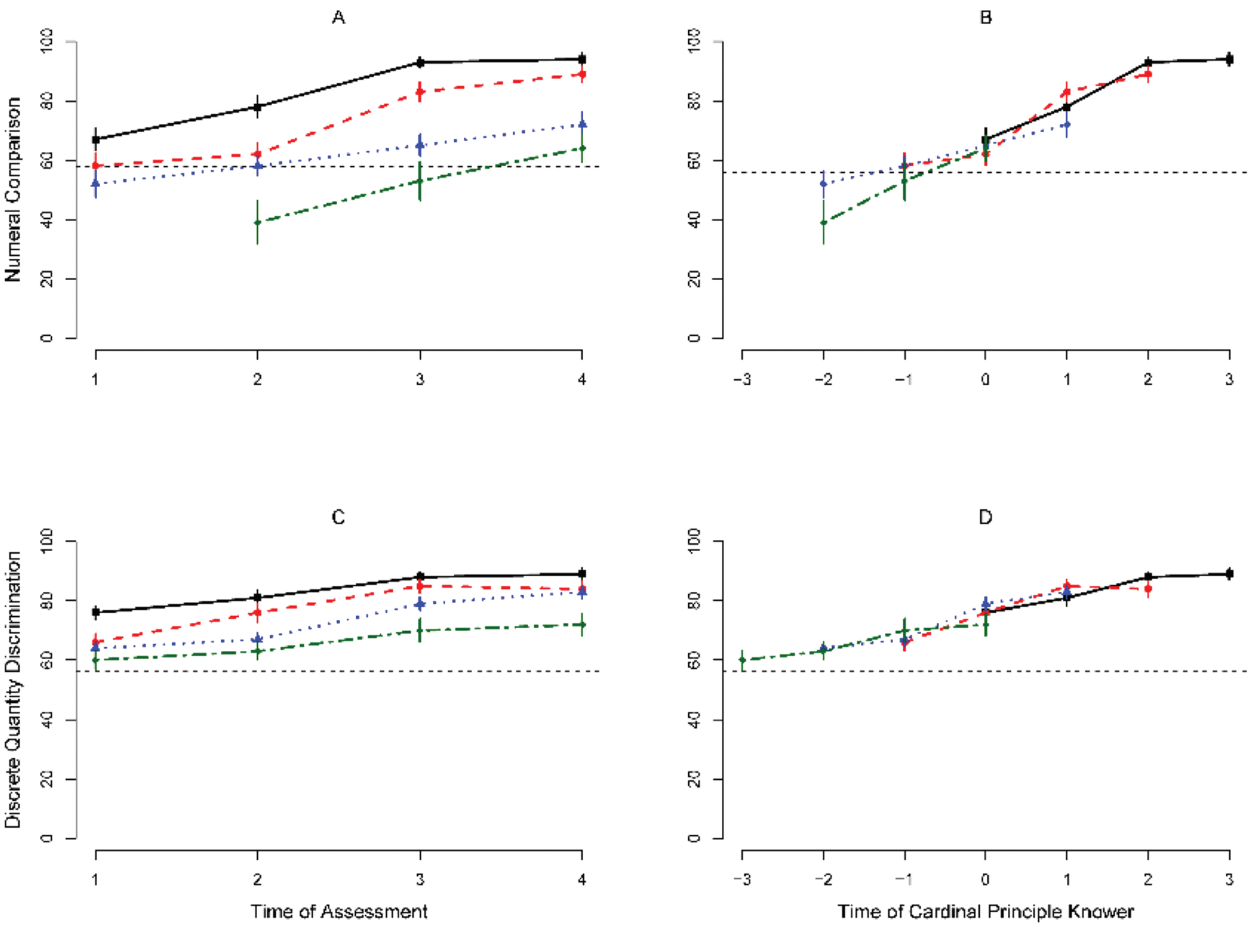


Examples of the type of items found on the number sets test. The goal is to quickly and accurately identify sets that match the target numeral. First graders' fluency on this measure predicts later risk of mathematics difficulties and long-term mathematics achievement, controlling other factors (e.g., parental income, intelligence).

The core findings are shown in Figure 3. The panels show four groups, children who were cardinal principle knowers at the beginning of preschool (black lines); those who achieved it before the end of the first year (red lines); those who were cardinal principle knowers at the beginning of the second year (blue lines); and those who achieved it during the second year (green lines). The top panels show performance on a numeral comparison task and the bottom performance on an ANS quantity discrimination task, as a contrast. The dashed lines are chance performance. The x-axis shows performance across the four preschool quantitative assessments (once a semester across the two years).

Panel C shows that children in all the groups were above chance on the ANS task, consistent with this being (at least in part) an inherent competency. In contrast, Panel A shows that children's understanding of the relative quantities represented by numerals varied by when they become cardinal principle knowers. The key results are shown in panels B and D, which aligns students based on when they became cardinal principle knowers, which is noted as 0. So, for the students who became cardinal principle knowers the second semester of the first year (red dashed line), -1 is the assessment before this (beginning of preschool) and assessments 1 and 2 are the two respective assessments in the second year. *The critical and unique finding here is that all group differences on the numeral comparison task disappear once they are aligned on the timing of becoming a cardinal principle knower; in fact, numeral comparison performance for all groups is at chance before this insight, whenever it occurred.* There were also gains on the ANS task following students becoming a cardinal principle knower, but these were less steep than those found for numerical comparison.

Figure 3:



Accuracy for performance on the numeral comparison and ANS tasks for groups that become cardinal principle knowers at the beginning of preschool (black, solid), during the first year of preschool (red, dashed), at the beginning of the second year (blue, dot), and during the second year (green, two dashed). Plots A and C show performance based on time of assessment and plots B and D show the same values aligned with the time the children became cardinal principle knowers (0 on *x*-axis): Negative values are assessment prior to becoming a cardinal principle knower and positive values are assessments after becoming a cardinal principle knower. Dash horizontal lines are chance performance. The figure is from *Growth of symbolic number knowledge accelerates after children understand cardinality* by D. C. Geary & K. van Marle, 2018, *Cognition*, p. 74.

The key point is that understanding the relations among numerals is a critical early foundation for formal math learning and building this foundation *cannot start* until students are cardinal principle knowers. Children who were cardinal principle knowers at the beginning of preschool had nearly a 2-year head start in learning the relations among numerals relative to children who did not attain it until the end of preschool (10% of the sample didn't attain it at all, not shown in Figure 3), which was reflected in their number system knowledge at the beginning of first grade.

Our next study was focused on identifying the factors that influence the age of becoming a cardinal principle knower.[27] Given that some children had achieved this status before starting preschool, it is reasonable to assume that home experiences were a contributing factor, which is discussed in the next section. For our study, the timing of children's cardinal principle status was predicted by their age, enumeration skills (i.e., the ability to use counting to determine the number of objects in a group), ANS acuity, letter recognition (which was very highly correlated with numeral recognition), and intelligence.

Among other factors, children who were cardinal principle knowers at the start of preschool had a longer count list ('one, two, three… up to one hundred'), knew more numerals, and were somewhat brighter (though still in the average range) than the other children. Children who did not achieve cardinal principle status before the end of preschool recognised only a few (if any) letters and numerals at the beginning of preschool, suggesting low exposure at home, and had low executive functions scores. The latter reflects the ability inhibit initial responses and switch from one task to another and then back to the first task: performance on these measures consistently predicts concurrent and later mathematics achievement.[28]

In all, our studies and related ones suggest that children's learning the cardinal values of number words and later numerals and becoming cardinal principle knowers before they are 4 to 4½ years old sets the foundation for school readiness. Becoming an early cardinal principle knower in turn appears to require the ability to enumerate (which requires knowing some count words), exposure to number activities at home, and the ability to stay focused.

Home Environment

The home environment, as noted, is a likely contributor to early individual differences in children's knowledge of the count list, ability to enumerate, and cardinality, but there is not a straightforward relation between home factors and these competencies. Cardinal knowledge is an evolutionary novel insight and thus we would not expect typical parent-child interactions to easily foster this knowledge.

Broadly, there are universal and likely evolved biases in parent-child interactions that facilitate the transmission of cultural knowledge.[29] The biases include children's attentiveness to and imitation of adult activities. The knowledge learned in this way tends to be instrumental — observable and repeatable activities resulting in a functional outcome — and focused on social conventions. Parental direct teaching of abstract concepts, such as cardinality, is unusual in traditional contexts and is sporadic even in developed ones.[30] For number development, parental engagement with young children would likely involve instrumental activities, such as counting objects, and the conveyance of cultural conventions, such as number words, and not the types of activities that are likely to promote cardinal principle knowledge.

Observation of parent-child number activities supports this expectation. The activities typically involve instrumental counting and use of simple number words, but frequency of engagement in these activities is often uncorrelated with children's later math achievement.[31] Positive relations often emerge when parents engage children in more advanced activities, such as comparing the cardinal values of numerals,[32] that may reflect an explicit goal of directly teaching school-related number knowledge.[33] Ramani and colleagues found that parental report of direct teaching correlated with basic (e.g., rote counting) and complex (e.g., cardinal knowledge) number skills, and that parent-child number talk that focused on concepts, such as cardinality, predicted complex but not basic number knowledge.[34]

Clearly, the relation between home numeracy experiences and early number development is nuanced and just encouraging parents to engage in more number activities with their children will not result in the activities that foster an early conceptual understanding of number. This is because when parents engage in academic related activities with their children they are biased to engage in instrumental activities (such as counting objects or learning the names of letters or numerals) and not activities that will push forward children's understanding of complex academic concepts, including cardinality. The development of complex number knowledge appears to occur when parents *explicitly* attempt to teach this knowledge and use it in their number talk. Even so, evidence for this relation is based on studies with small sample sizes and without a broad assessment of parental characteristics that might influence these activities. Further studies are needed to determine the specific parent-child activities and parent (e.g., their math knowledge) and child (e.g., attentional control) factors that foster the early emergence of cardinal principle knowledge and supporting competencies (i.e., the count list, and enumeration).

Implications

These studies imply that early interventions are needed for children who are at risk for long-term difficulties with mathematics. Indeed, there are several broad-based mathematics interventions for preschoolers that improve overall mathematics achievement,[35] but these generally suffer from fade-out effects, that is, the intervention benefits fade after a year or two.[36] A double-dose of these interventions may prevent some of these fade-out effects,[37] but this remains to be fully evaluated.

An analogous pattern of fade-out is found with individual therapy for juvenile offenders but sustained long-term benefits can be achieved with multisystemic approaches that simultaneously involve individual-, home-, and school-based interventions.[38] These multisystemic interventions are based on a substantive literature on the family dynamics, school behaviour, and peer relationships of these children and adolescents.[39] In other words, it is likely that better preparing at-risk students for formal mathematics learning at school entry will require a multi-systemic intervention.

Along with a multi-systemic approach, our studies suggest that more targeted interventions focusing on counting, enumeration, and cardinal knowledge might be effective, but this too remains to be determined. Moving forward will involve identifying the parent-child interactions, preschool-classroom activities, and child-centered (e.g., focusing on executive and attentional control) factors that promote the development of these competencies. An associated multi-systemic intervention would involve simultaneously intervening in all these areas.

Conclusions

Much is known about infants' and young children's quantitative knowledge and development, but the bridge between these competencies and their later readiness for and performance in school mathematics is not well understood. The studies that have been conducted suggest that risk of long-term difficulties with mathematics — or at least starting school significantly behind ones' peers in fundamental numerical knowledge — can be determined by 3½ to 4 years of age by the length of children's count list (how far they can count without error), their ability to use counting to determine the number of objects in a group, and their understanding of the cardinal values represented by number words. In fact, performance on just the give-a-number task provides considerable information about risk, with children who do not know the cardinal value of any number words or are only one-knowers or two-knowers at 4 years of age being at high risk. Not surprisingly, performance on these counting, enumeration, and cardinality tasks is highly correlated with performance on standardised mathematics achievement tests designed for preschoolers,[40] but the former are more directly diagnostic of the skills and knowledge that need to be addressed than is performance on the latter.

At the same time, we do not yet have a complete understanding of the home (e.g., nature of parent-child number talk), school (e.g., number activities in preschool classrooms), and child (e.g., attentional control) factors that influence the development of this core number knowledge. A fuller understanding of these factors and how they interact will provide a solid foundation for the development of interventions for young children who are at-risk for long-term difficulties in mathematics.

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